Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero vectors such that no two of them are collinear and $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$. If θ is the angle between vectors $\vec{b}$ and $\vec{c}$, then a value of $\sin\theta$ is :

(1) $\frac{2}{3}$

(2) $\frac{-2\sqrt{3}}{3}$

(3) $\frac{2\sqrt{2}}{3}$ ← Correct answer

(4) $\frac{-\sqrt{2}}{3}$

$$(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{c} \cdot \vec{a})\vec{b} - (\vec{c} \cdot \vec{b})(\vec{a})$$

Given $(\vec{a} \times \vec{b}) \times \vec{c} = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

Equating

$$(\vec{c} \cdot \vec{a})\vec{b} - (\vec{c} \cdot \vec{b})(\vec{a}) = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

Since $\vec{a}$ & $\vec{b}$ are not collinear, we can write

$$-(\vec{c} \cdot \vec{b})(\vec{a}) = \frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$$

$$\Rightarrow -(\vec{c} \cdot \vec{b}) = \frac{1}{3}|\vec{b}||\vec{c}|$$

$$\Rightarrow \cos\theta = \frac{1}{3}$$ where θ is the angle between $\vec{b}$ & $\vec{c}$

$$\therefore \sin\theta = \sqrt{1-\cos^2\theta} = \pm\frac{2\sqrt{2}}{3}$$

Option (3) is the correct answer.