A complex number z is said to be unimodular if $|z|=1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1 \bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a :

(1) circle of radius 2.

(2) circle of radius $\sqrt{2}$.

(3) straight line parallel to x-axis.

(4) straight line parallel to y-axis.

$$\left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right| = 1$$

$$\Rightarrow \left|\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right|^2 = 1$$

$$\Rightarrow \left(\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right)\left(\overline{\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}}\right) = 1$$

$$\Rightarrow \left(\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}\right)\left(\frac{\bar{z}_1 - 2\bar{z}_2}{2 - \bar{z}_1 z_2}\right) = 1$$

$$\Rightarrow \left\{|z_1|^2 - 4\right\}\left\{1 - |z_2|^2\right\} = 0$$

Since $|z_2| \neq 1$

$$\Rightarrow \underbrace{|z_1| = 2}$$

Circle with center $(0,0)$ and radius 2

$\therefore$ Correct option is (1)