Let $y(x)$ be the solution of the differential equation

$(x \log x)\dfrac{dy}{dx} + y = 2x \log x, (x \geq 1)$.

Then $y(e)$ is equal to :

(1) 2 ✓

(2) 2e

(3) e

(4) 0

DOPREP

$$(x \log x)\frac{dy}{dx} + y = 2x \ln x$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x \log x} = 2 \quad \text{----} \ (1)$$

$$I.F. = e^{\int \frac{1}{x \log x} dx} = e^{\log(\log x)} = \log x$$

$\therefore$ The solution of (1) is

$$y(\log x) = \int 2 \log x \, dx$$

$$\Rightarrow y(\log x) = 2x \ln x - 2x + C \quad \text{---} \ (2)$$

We have $(x \log x)\frac{dy}{dx} + y = 2x \log x$, then we get that, when $x = 1$; $y = 0$

Replacing this condition in (2)

we get $C = 2$

$\therefore$ Solution is

$$y(\log x) = 2x \ln x - 2x + 2$$

For $x = e$, we get

$y = 2e - 2e + 2 = 2$

$\therefore$ Correct option is (1)