If m is the A.M. of two distinct real numbers l and n ($l, n > 1$) and G_1, G_2 and G_3 are three geometric means between l and n, then $G_1^4 + 2G_2^4 + G_3^4$ equals.

(1) $4\, lmn^2$

(2) $4\, l^2m^2n^2$

(3) $4\, l^2mn$

(4) $4\, lm^2n$ ✓

DOPREP

$$m = \frac{l+n}{2} \quad \text{----} (1)$$

l, G_1, G_2, G_3, n are in G.P

$\therefore G_1^2 = l G_2 \; ; \; G_2^2 = G_1 G_3 \; ; \; G_3^2 = n G_2$

$$G_1^4 + 2G_2^4 + G_3^4 = l^2 G_2^2 + 2 G_1^2 G_3^2 + n^2 G_2^2$$

$$= G_2^2 (l^2 + 2nl + n^2)$$

$$= G_2^2 (2m)^2 = 4m^2 G_2^2 \quad \text{----} (3)$$

We know that $G_2^2 = G_1 G_3 = \sqrt{nl}\, G_2$

$$\Rightarrow G_2 = \sqrt{nl}$$

$\therefore$ Equation (3) becomes

$$G_1^4 + 2G_2^4 + G_3^4 = 4m^2 nl$$

$\therefore$ Correct option is (4)