

Let

$$\tan^{-1} y = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

where $|x| < \frac{1}{\sqrt{3}}$. Then a value of y is:

(1) $\frac{3x - x^3}{1 + 3x^2}$

(2) $\frac{3x + x^3}{1 + 3x^2}$

✓ (3) $\frac{3x - x^3}{1 - 3x^2}$

(4) $\frac{3x + x^3}{1 - 3x^2}$

$$\tan^{-1} y = \tan^{-1} x + \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

$$= \tan^{-1} \frac{x + \frac{2x}{1-x^2}}{1 - \frac{2x^2}{1-x^2}} = \tan^{-1} \frac{x - x^3 + 2x}{1 - x^2 - 2x^2}$$

$$= \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right)$$

∴ A value of y can be $\frac{3x - x^3}{1 - 3x^2}$

∴ Correct option (3)