Let $f(x)$ be a polynomial of degree four having extreme values at $x = 1$ and $x = 2$.

If $\lim_{x \to 0}\left[1 + \frac{f(x)}{x^2}\right] = 3$, then $f(2)$ is equal to :

(1) 0
(2) 4
(3) -8
(4) -4

DOPREP

$$f(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$$

$$\therefore f'(x) = 4Ax^3 + 3Bx^2 + 2Cx + D$$

given $f'(1) = f'(2) = 0$

$$\therefore 4A + 3B + 2C + D = 0 \text{ ------ (i)}$$

$$32A + 12B + 4C + D = 0 \text{ ------ (ii)}$$

$$\underset{x \to 0}{Lt}\left[1 + \frac{Ax^4 + Bx^3 + Cx^2 + Dx + E}{x^2}\right] = 3$$

$$\Rightarrow \underset{x \to 0}{Lt}\left(1 + Ax^2 + Bx + C + \frac{D}{x} + \frac{E}{x^2}\right) = 3$$

For the limit to exist $\rightarrow$ $\boxed{\therefore D = E = 0}$

$$\Rightarrow 1 + C = 3 \Rightarrow \boxed{C = 2}$$

Replacing in (i) & (ii) we get

$$4A + 3B = -4$$

$$32A + 12B = -8$$

Solving $\rightarrow$ $\boxed{A = \frac{1}{2} ; B = -2}$

$\therefore$ The polynomial is

$$f(x) = \frac{x^4}{2} - 2x^3 + 2x^2$$

$$\therefore f(2) = 8 - 16 + 8 = 0$$

Correct option is (1)