13. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its center and perpendicular to one of its faces is :

(1) $\frac{4MR^2}{9\sqrt{3}\pi}$ ← correct

(2) $\frac{4MR^2}{3\sqrt{3}\pi}$

(3) $\frac{MR^2}{32\sqrt{2}\pi}$

(4) $\frac{MR^2}{16\sqrt{2}\pi}$

The cube of max volume will have its longest diagonal as the diameter of the sphere

Longest diagonal $= \sqrt{3a^2} = 2R$

$\therefore a = \frac{2R}{\sqrt{3}}$ 'a' is side of the square

Also, density of sphere ρ is

$\rho = \frac{M}{4\pi R^3}$

$\therefore$ mass of cube $= \rho a^3 = \frac{2M}{\sqrt{3}\pi} = m$

Moment of inertia of cube

$I = \frac{ma^3}{6}$

$\therefore I = \frac{2M}{\sqrt{3}\pi} \frac{1}{6} \left(\frac{2R}{\sqrt{3}}\right)^3$

$I = \frac{4MR^2}{9\sqrt{3}\pi}$ (1) is correct option